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An analysis of flow through sudden enlargements in pipes

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An integral analysis of the type used to predict the flow of co-flowing jets has been applied to the problem of a sudden enlargement in a pipe (Borda–Carnot expansion). This technique successfully predicts all the overall flow parameters of interest (e.g. reattachment lengths, pressure profile, etc.). The analysis indicates that the downstream conditions (up to reattachment) are insensitive to wall shear and the point of minimum pressure does not coincide with the location of the maximum return-flow velocity.

1. Introduction

Separation in internal flow has been a common occurrence in engineering either by design or default every since fluids have been transported in closed conduits. Perhaps the most common device which exhibits separation is a sudden enlargement (expansion) in a pipe. The sudden enlargement is among those devices commonly found in piping systems which have received the least analytic attention. The first analysis of sudden-enlargement flows was made by Borda (1766) and since that time the literature has concerned itself with the confirmation of his expression for the overall loss. The loss equation that Borda derived was part of a more involved problem concerning the draining of vessels.

An example of a recent confirmation of Borda's equation is the extensive experimental work carried out by Lipstein (1962). This series of tests was conducted for the range of beta ratios (the ratio of the upstream pipe diameter to the downstream pipe diameter) 0·133–0·9 and for Reynolds numbers up to $2 \cdot 6 \times 10^5$. The experimental set-up provided for a thin boundary layer at the entrance to the expansion, which simulates the conditions for which the Borda loss equation was derived. The most interesting aspect of the study was the axial pressure gradient along the wall. This gradient was initially negative for some distance beyond the enlargement and thereafter became positive until the pressure rose to within a few per cent of the Borda value. Similar behaviour may be found in other separated-flow situations (orifices, stalled diffusers, mitre-bends, etc.). Lipstein was primarily concerned with the gross effects of the enlargement and measured only a few velocity profiles at scattered locations downstream of the enlargement. Characteristic of these profiles near the beginning of the enlargement was

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the flatness of the velocity profile, which was also found by an earlier investigator, Kalinske (1944). Kalinske found that within this 'potential core' the velocity profile was flat and the turbulent intensity was no higher than that in the upstream pipe. It was also found that this core rapidly disappeared within a few pipe diameters of the beginning of the enlargement.

More recently the existence of the core was reconfirmed by Chaturvedi (1963) using hot wires. This paper is perhaps the most comprehensive exposition of the flow field downstream of an enlargement. In addition to the 'potential core' with its flat velocity profile, the sudden enlargement was found to have a flat static pressure profile in the radial direction, and an extensive recirculation zone. Along the edge of the core and the recirculation zone a region of high turbulent shear stress was found, not unlike that found for co-flowing jets. Chaturvedi found that this behaviour also occurred in highly stalled diffusers.

The flow in sudden enlargements in pipes has been found to have several similarities with co-flowing jets and the flow behind bluff bodies (i.e. a potential core, shear region and secondary-flow region); these similarities are sufficient such that the techniques developed to analyse the latter flows may be used to solve the sudden-enlargement problem. These internal jet (or jet-like) problems have generally been solved using integral techniques. At present two such techniques (involving moments of momentum and integration of the momentum equation to different upper limits respectively) are in common use. Examples of the application of the moments-of-momentum method may be found in the work of Hill (1965, 1967) for co-flowing jets and Narayanan (1972) for the flow downstream of a leaf gate. The multi-upper-limit method was recently developed by Bowlus, Rogers & Brighton (1969) and has yielded results little, if at all, different from those of the moment method for the co-flowing jet problem. In the above solutions the pressure gradient is usually replaced by a characteristic velocity gradient (Hill 1965; Bowlus *et al.* 1969), neglected entirely within the separated region (Hill 1967) or replaced by an empirical pressure gradient (Narayanan 1972).

Various velocity profiles have been used by the above authors without any discernible difference in their results. Both Abramovich (1963, p. 393) and Bowlus *et al.* favoured a profile of the form

$$U = U_2 - (U_2 - U_1)(1 - \eta^{1\cdot5})^2.$$

In Abramovich's solution for the flow downstream of a bluff body in the centre of a pipe, U_1 was the primary-flow velocity and U_2 the return (or upstream) velocity in the recirculation zone behind the obstruction. There are some striking differences between Abramovich's solution and the aforementioned methods. He integrated the momentum equation only once and integrated the continuity equation twice to form the required number of independent equations. In addition he neglected entirely the effects of the shear stress.

The more standard techniques for the solution of internal flow problems rely on empirical information for the calculation of the turbulent shear stress in the mixing or shear region. In the case of jet flows the eddy viscosity is given by

$$\epsilon = kb(U_{\max} - U_{\min}),$$

where b is the shear-region width and k is an empirical constant. The value of k is by no means universal in the literature for co-flowing jets. In general the range of k is found to be 0·006–0·02, which can be accounted for by the selection of the assumed velocity profile.

2. Method of solution

Sudden-enlargement flows are somewhat more complex than the flow of co-flowing jets as the separated region must be treated from the beginning of the enlargement. The basic flow may be subdivided into at least three distinct axial zones which must be idealized for the analytic treatment of the problem. The first zone consists of the area immediately following the enlargement. In this zone, surrounding the centre of the pipe is the 'potential core' where the turbulent intensity is low. Moving radially outward, a region of high turbulent shear exists which consists of both downstream flow and a portion of the recirculation eddy. As the wall is approached the turbulent shear $\overline{u'v'}$ decreases to zero and the flow becomes not unlike that towards a step. Further downstream from this zone there are two possible flow patterns which may occur at the centre of the pipe depending on the beta ratio. At low to moderate betas ($< 0{\cdot}6$) the 'potential core' is completely eradicated and the downstream flow approaches the same profile as is found in free jets, where the centre-line velocity gradually diminishes. Within the recirculation eddy the flow pattern remains similar to that found further upstream near the start of the enlargement. However, the velocity in the upstream direction has reached its peak and also begins to decrease. This flow pattern continues until reattachment occurs. For the larger beta ratios the 'potential core' may exist well beyond the reattachment point, with the return flow following the same pattern as described above. The third flow zone (which was not analysed) occurs beyond the reattachment point where the velocity profile continues to change until a fully developed turbulent pipe flow profile is achieved.

From these basic patterns for sudden-enlargement flows a simplified model was established. Figure 1 (a) represents the idealization of the flow for low betas and figure 1 (b) that for high beta ratios. Turbulent shear was considered to be important only from R_1 to R_2 in zone A, from R_1 to the wall in B, from the centre-line to R_2 in C and across the whole pipe in D. For the return-flow velocity it is necessary to reverse the problem and look at the physical situation from the point of reattachment back to the point at the beginning of the enlargement. From this viewpoint the initial pressure gradient is very favourable and thus the boundary layer does not grow rapidly until the flow approaches the upstream wall. Rather than attempting to model the boundary layer the jet profile was extended to the pipe wall. In zones A and C the boundary layer should start to show some growth as the pressure gradient changes from negative to positive (looking upstream), but since turbulent boundary layers are generally flat it was assumed that a flat velocity profile would suffice.

The extensive data compiled by Chaturvedi form an excellent basis for some observations which considerably simplify the equations of motion for a sudden

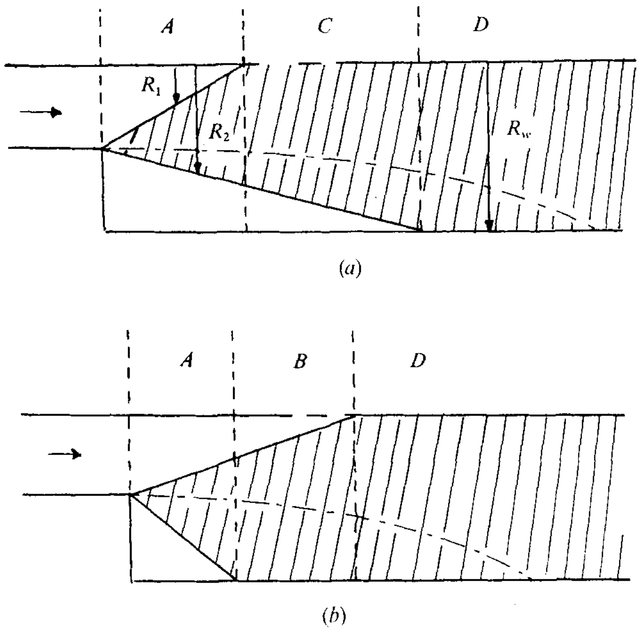

FIGURE 1. Idealization of sudden-enlargement flows: R_1 = radius of jet potential core; R_2 = inner radius of constant-velocity return flow. Region of influence of turbulent shear stresses is cross hatched. (*a*) Low to moderate beta ratios (< 0·6). (*b*) High beta ratios. *A*, transition zone; *B*, attached transition zone; *C*, fully established jet-flow zone; *D*, fully established attached-flow zone; —·—, separation streamline.

enlargement. In general the available data indicate that the radial variation of the Reynolds stress is much greater than the axial, the radial pressure gradient is very nearly zero and the radial velocity and its derivatives are small compared with the axial velocity and its derivatives. Neglecting any viscous stresses (except at the wall), the equations of motion for the sudden enlargement can then be written as

$$\frac{1}{r}\frac{\partial(rV)}{\partial r}+\frac{\partial U}{\partial x}=0, \tag{1}$$

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{r}\frac{\partial(r\tau)}{\partial r}. \tag{2}$$

If these equations are non-dimensionalized using the initial centre-line velocity and the enlargement pipe radius the integral equations are

$$\frac{d}{dx}\int_0^1 Ur\,dr=0 \quad \text{(continuity)}, \tag{3}$$

$$\frac{d}{dx}\int_0^1 U^2r\,dr=-\frac{1}{2}\frac{dp}{dx}+C_f \quad \text{(momentum to the wall)}, \tag{4}$$

$$\frac{d}{dx}\int_0^r U^2r\,dr-U_r\frac{d}{dx}\int_0^r Ur\,dr=-\frac{r^2}{2}\frac{dp}{dx}-r\overline{u'v'} \quad \text{(momentum to } r\text{)}. \tag{5}$$

Further simplification of the solution to these equations involves the assumption of self-preservation. Self-preservation of the mean velocity profiles has been shown to be very nearly true for the case of co-flowing jets by Curtet & Ricou (1964). Because of the other similarities between co-flowing jets and the sudden enlargement the assumption of self-preservation was used in this analysis. Several velocity profiles may be chosen, all of which will yield approximately the same results. For this analysis the velocity profile previously cited was used as the basis for the enlargement profile. For this case U_1 represents the maximum forward velocity and U_2 the maximum upstream velocity. Application of the above idealizations resulted in the appearance of four zones of possible flow (see figure 1).

In the transition zone (A in figure 1) where the expansion begins the velocity profile near the centre-line is flat and gradually decays to a fully developed jet profile. Within this zone the velocity profile is described by a jet core from 0 to R_1 in which

$$U = U_1,$$

a shear region from R_1 to R_2 (i.e. of width b) in which

$$U = U_2 - (U_2 - U_1)(1 - \eta^{1.5})^2,$$

where $\eta = (r - R_1)/b$, and a wall region from R_2 to R_w ($= 1$) in which

$$U = U_2.$$

Depending on the beta ratio, the outer edge of the shear layer may 'attach' to the wall before the core has decayed. This, of course, cannot happen in a real flow as a boundary layer is always present. For this attached transition zone (B in figure 1), the velocity profile takes the form of a jet core from 0 to R_1 in which

$$U = U_1$$

and a shear region from R_1 to R_w (equal to 1 for integration) in which

$$U = U_2 - (U_2 - U_1)(1 - \eta^{1.5})^2,$$

where $\eta = (r - R_1)/b$.

When the core has decayed the velocity profile takes the form of a fully developed jet. For this zone, of fully established jet flow (C in figure 1), the profile takes the form of a shear region from 0 to R_2 in which

$$U = U_2 - (U_2 - U_1)(1 - \eta^{1.5})^2,$$

where $\eta = r/R_2$, and a wall region from R_2 to R_w in which

$$U = U_2.$$

The final zone in the analysis is that of fully established attached flow (D in figure 1). By this point the core has decayed and has a profile similar to that for a free jet, and the edge of the shear layer has 'attached' (it should be noted that at the true reattachment point the shape of this profile is correct as this would be a point of zero shear stress). The profile takes the form of a shear region from 0 to R_w in which

$$U = U_2 - (U_2 - U_1)(1 - \eta^{1.5})^2.$$

The turbulent shear stress within the shear region was calculated using the assumed profile in the equation

$$-\overline{u'v'} = kb(U_1 - U_2)\,\partial U/\partial r. \tag{6}$$

The value of k which gave the most consistent results for the range of beta ratios that were of interest was 0·0087. By choosing this value for the constant, the calculated values of $\overline{u'v'}$ were found to be within a few per cent of the measured maximum values (Chaturvedi 1963). This value of k is equivalent to a turbulent Reynolds number Ub/ϵ of approximately 114. At the wall the shear stress was assumed to follow the relation $C_f = C_1 U_2^2$. This form was chosen as it represents the apparent path of the wall shear stress, i.e. from zero at the start of the expansion to a maximum, and then back to zero at the reattachment point.

Both Hill and Narayanan analysed cases (other than the sudden enlargement) where recirculation zones existed. In their analyses they either neglected the pressure gradient or used an empirically derived pressure distribution in the analysis. Heskestad (1970), Lipstein and Chaturvedi have found that, within a recirculation zone of an expansion, the pressure does not remain constant but initially decreases, then increases above the value found at the origin of the expansion. Where the 'potential core' exists the pressure is generally below the initial value. Heskestad postulates that in this region one can consider the jet to behave as a free jet. Rather than defining the decay rate of the jet core it is more suitable to define the growth of the shear layer. For similar situations Abramovich (1963, p. 393) has chosen a simple linear function for the expansion of this layer, the form being $b = cx$. The data compiled by Chaturvedi indicate that a value of the constant in the neighbourhood of 0·35 eliminates (in the analysis) the influence of turbulent shear stresses whose magnitudes are less than 5 % of the maximum at any given axial location.

The profiles shown above were substituted into both the continuity equation and the momentum equations, resulting in a closed system of nonlinear ordinary differential equations for each of the four zones. These equations were of the form

$$a_j \frac{dU_1}{dx} + b_j \frac{dR_1}{dx} + c_j \frac{db}{dx} + d_j \frac{dU_2}{dx} + e_j \frac{dp}{dx} = f_j, \tag{7}$$

where the $a_j, \ldots$ are the coefficients which result from the integration and subsequent differentiation (these coefficients are numerous, complex and readily available in the work of Teyssandier (1973), and therefore will not be presented here). The above set of equations was used to calculate the results for sudden-expansion flows presented in the following section.

3. Results of the analysis

The method derived in the previous section provided several important flow parameters which could be compared with the existing data. The static pressure along the pipe wall is the most widely available of the quantities which this model can predict as it is the easiest to measure. A sample of the predicted

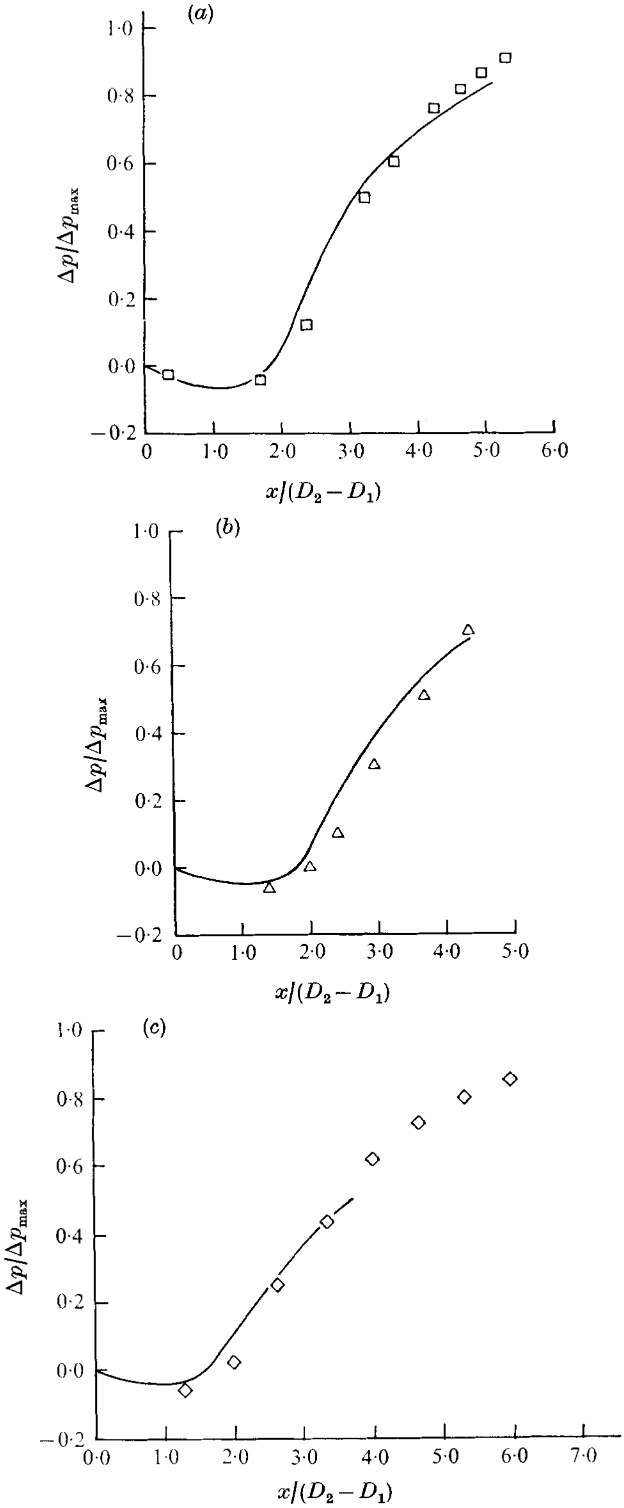

FIGURE 2. Predicted axial pressure profiles (solid lines, local Δp/Borda–Carnot Δp). Axial distance x normalized by enlargement diameter difference $D_2 - D_1$. (*a*) $\beta = 0{\cdot}4$; □, experiment, Lipstein. (*b*) $\beta = 0{\cdot}5$; △, experiment, Ackeret (1967). (*c*) $\beta = 0{\cdot}625$; ◇, experiment, Lipstein.

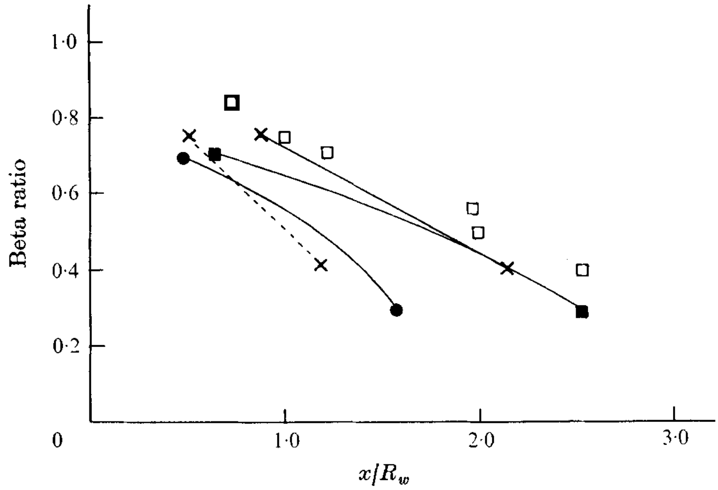

FIGURE 3. Axial locations of pressure minima and zero-Δp points (pressure recovers to value at beginning of enlargement). Pressure minima: ●—●, calculated; ×----×, experiment, Lipstein. Zero-Δp points: ■—■, calculated, ×—×, experiment, Lipstein; □, experiment, Ackeret.

pressure profiles (divided by the Borda loss $\beta^2 - \beta^4$) for several beta ratios may be found in figures 2(*a*)–(*c*). The axial location was non-dimensionalized by the diameter difference of the enlargement in an attempt to scale the expansion. In comparing the three figures this has a tendency to match the axial locations of pressure minima and zero-pressure points, but not the reattachment points. The pressure profiles show excellent agreement between the data and the predictions. The model does not predict the maximum pressure rise downstream of the expansion as it is valid only up to the reattachment point, and further modification of the velocity profile must take place. Near the reattachment point the predicted pressure is slightly lower (though still acceptable) than the data. Since the governing equations were of a boundary-layer type it is questionable whether they may be sufficient to define completely the flow around this location. Two other authors (Hill 1967; Narayanan 1972) who have analysed other recirculation zones use the same set of equations, but as they did not directly calculate the pressure it is not possible to determine whether their methods would show the same effect.

This model also makes predictions of the axial location of the point of minimum pressure and the point where the pressure recovers to the value at the start of the enlargement (zero-Δp point). The results from the analysis (see figure 3) are in good agreement with the available experimental locations. The reattachment point, or the point where the jet diffuses to the wall, is another important parameter in this type of flow. Although attachment data are limited for this range of beta ratios there are sufficient data to show that the results (see figure 4) of the analysis fall within the data scatter.

Table 1 is an outline of the end points of the flow zones defined in figure 1 for the range of beta ratios considered. For high to moderate beta ratios, the

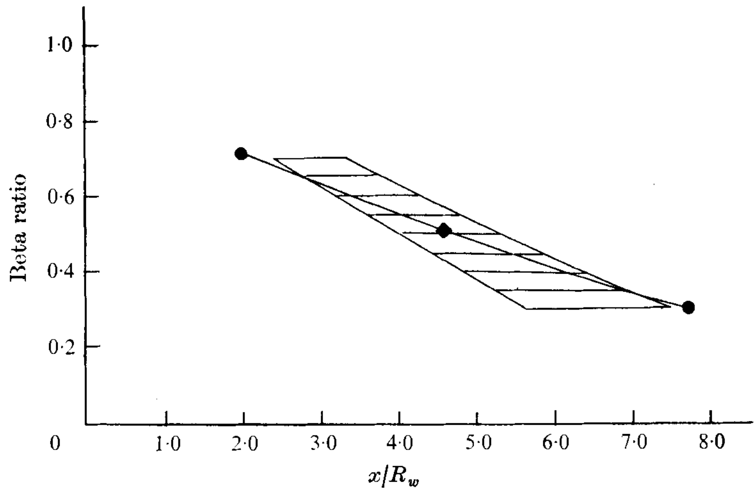

FIGURE 4. Jet reattachment points for various enlargements. ●—●, calculated; ◆, experiment, Chaturvedi; ☰, experiment, Lipstein.

	$\beta = 0{\cdot}7$	$\beta = 0{\cdot}6$	$\beta = 0{\cdot}5$	$\beta = 0{\cdot}4$	$\beta = 0{\cdot}3$
Transition (A)	2·78 [1·67]	2·43 [1·82]	2·12 [2·12]	2·01 [2·41]	1·65 [2·31]
Attached transition (B)	3·32 [1·99]	3·73 [2·80]	4·15 [4·15]	2·63 [3·16]	—
Fully established jet (C)	—	—	—	—	1·85 [2·58]
Fully established attached flow (D)	—	—	4·36 [4·36]	5·02 [6·02]	5·50 [7·70]

TABLE 1. Values of $x/(D_2-D_1)$ $[x/R_w]$ at end points of flow zones.

shear layer 'reattaches' before the core collapses. Likewise for these enlargements pressure minima and zero pressure differences occur within the transition zone. At the highest beta ratios (0·7 and 0·6) the core had not yet decayed even at the point of reattachment. As would be expected this attachment point occurs further downstream for the lower beta ratios since the diameter difference is larger for these cases. Conversely the core decay occurs further upstream as the beta ratio is decreased (at the lower beta ratios one finds that the core decays before the shear layer attaches). A similar pattern occurs in free jets, but the confined jet should decay faster since the turbulent shear stresses are higher.

For the internal structure of the flow two comparisons can be made with the data of Chaturvedi. One of these, of the return-flow velocity within the recirculation zone, is shown in figure 5. Narayanan's model of the flow downstream of a leaf gate showed the same tendency to overpredict this quantity. Compared with his analysis, the present calculations agree reasonably well with the data. The other comparison which can be made is of the shape of the separation streamline as presented in figure 6. Here there is substantial agreement with the measured shape. Predictions were made for all cases considered, but owing to the lack of data for cases other than that shown no other checks could be made. A point of interest concerning the peak velocity in the recirculation zone is that it does not coincide (in either the data or the analysis) with the point of minimum

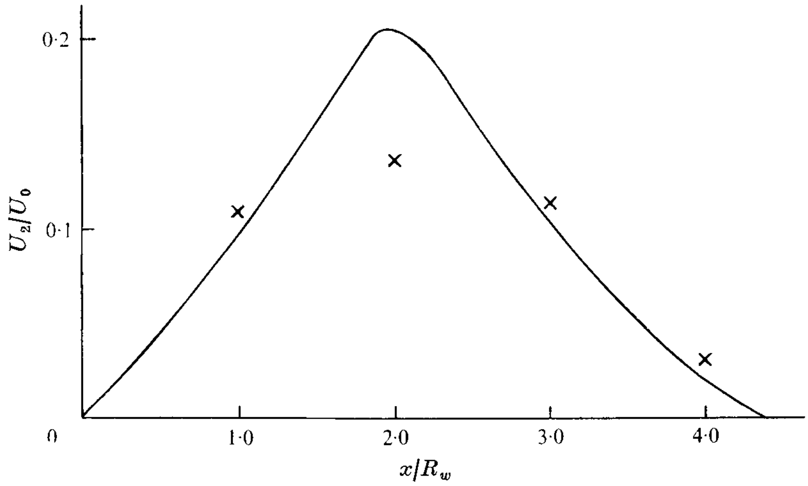

FIGURE 5. Return-flow velocity for $\beta = 0{\cdot}5$. ——, calculated; ×, experimental maximum (upstream) velocities, Chaturvedi.

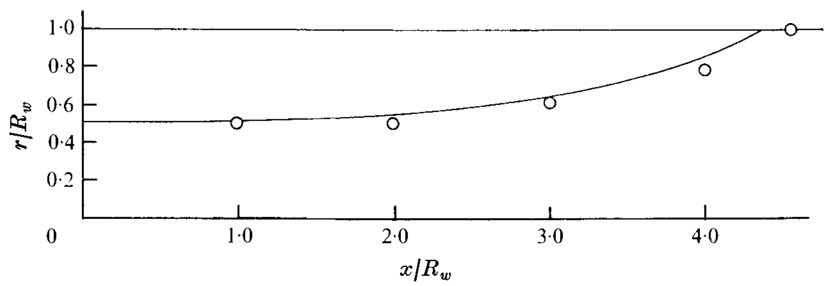

FIGURE 6. Shape of the separation streamline for $\beta = 0{\cdot}5$. ——, calculated; ○, measured mean location, Chaturvedi.

pressure. Unfortunately this does not lend itself to a simple explanation for the pressure decrease immediately downstream of the expansion.

As in other analyses of this type certain empirical constants are left to be chosen. Those chosen for this analysis were based on the available observed data and the reasons for their choice were stated in a previous section. It was found for the spread parameter and the constant for the calculation of the turbulent shear stress that variations of 6–10 % in the chosen values could alter the results. Since none of the surveyed papers mentioned the effect of varying parameters no comparisons can be made. It was found that it was possible to vary the constant for the calculation of the wall shear stress from 0 to 0·1 without any significant effect whatsoever. Hill (1967) found this to be true for co-flowing jets, but there is no experimental verification of this available.

4. Conclusions

The results of the analysis presented indicate that a method that has been successfully applied to the solution of co-flowing jets may also be used with suitable modification to analyse the sudden-enlargement problem. The analysis demonstrates that the full pressure recovery does not occur at the point of reattachment and may not occur for several more diameters downstream of that point. It is therefore necessary to allow sufficient length downstream of the enlargement in order to receive the benefit of the full pressure recovery. This analysis implies that the roughness (which controls the wall shear) should have little effect on the pressure gradient up to the reattachment point and should not significantly affect the reattachment length.

As with all internal-jet solutions there is a certain degree of freedom with regard to the selection of the empirical constants which govern turbulent parameters. The turbulent Reynolds number based on the data is of about the same order of magnitude as those which have been used in other analyses, but significant deviation from this value will alter the results. In regard to the boundary-layer assumptions that were made there is some question as to their validity (in spite of their frequent use for this type of analysis) in the vicinity of the reattachment point as the results begin to deviate from the measured values in this area.

Considering the numerous applications of the sudden enlargement (and its different flow phenomena) it is somewhat surprising that so little information exists about this basic flow device. Since it has been demonstrated here that this is a tractable problem which does lead to some important questions regarding pressure gradients it is hoped that the availability of data will increase in the future.

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